Parity, Clumpiness and Rational Choice

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Some philosophers believe that two objects of value can be 'roughly equal', or 'on a par', or belong to the same 'clump' of value in a sense that is fundamentally different from that in which some objects are 'better than', 'worse than', or 'equally as good as' others. This article shows that if two objects are on a par, or belong to the same clump, then an agent accepting a few plausible premises can be exploited in a money-pump. The central premise of the argument is that value is choice-guiding. If one object is more valuable than another, then it is not permitted to choose the less valuable object; and if two objects are equally valuable it is permitted to choose either of them; and if two objects are on a par or belong to the same clump it is also permitted to choose either of them.

I. INTRODUCTION

Who was the better philosopher, Jean-Paul Sartre or George Edward Moore? Moore did not write as elegantly as Sartre, but his philosophical ingenuity was exceeded by none. In the terminology introduced by Ruth Chang, we may say that Sartre and Moore were 'on a par' with respect to their philosophical skills.[1] Two objects are on a par if and only if (i) they are comparable, and (ii) it is false that one is better than the other, and (iii) it is false that they are equally good. Parity is thus a unique, fourth positive value relation, which differs fundamentally from the traditional value relations 'better than', 'worse than' and 'equally as good as'.[2] Derek Parfit's and James Griffin's concept of 'rough equality' is designed to capture essentially the same intuition about evaluative comparisons between very disparate items.[3]

In an article published in a recent issue of this journal, Nien-he Hsieh challenged the concept of parity. He did so by introducing an alternative concept of 'clumpiness', which on his view gives a better account of evaluative comparisons between disparate items. According to Hsieh, the notion of clumpiness 'sorts items into classes, or clumps, based upon the degree to which the items possess each of the relevant

[1] Ruth Chang, 'The Possibility of Parity', *Ethics* 112 (2002), pp. 659–88.

[2] Erik Carlsson, 'Parity Defined in Terms of Betterness', *Hommage à Wlodek: Philosophical Papers Dedicated to Wlodek Rabinowicz*, ed. T. Ronnow-Rassmussen et al. (Lund, 2007), argues that Chang's concept of parity can be defined in terms of betterness. Carlson's definition is interesting, but for our present purposes it does not matter whether it captures Chang's original intentions or not.

[3] Derek Parfit, *Reasons and Persons* (Oxford, 1984), p. 431; James Griffin, *Well-Being* (Oxford, 1986), pp. 96–8.

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respects that compromise the covering consideration'.[4] For example, Sartre and Moore belong to the same clump, meaning that even though very different, their philosophical skills were equally good. The main difference between parity and clumpiness is that the latter concept, but not the former, is consistent with the Trichotomy thesis, holding that whenever two items are comparable exactly one of the three relations 'better than', 'worse than', or 'equally as good as' obtains between the objects.

Hsieh argues that a strong reason for adopting concepts such as parity and clumpiness is that 'The incomparability of two items is thought to pose a problem for making justified choices and for consequentialist theories that rely on comparing states of the world.'[5] However, on Hsieh's view, if two items turn out to be on a par or belong to the same clump, rather than being incomparable, rational choice theory could be applied for deciding between them. In this article I argue that, contrary to Hsieh's view, there is no way of linking either parity or clumpiness to rational choice. More precisely put, this article shows that *if* two items are on a par, or belong to the same clump, then an agent accepting a few plausible premises can be exploited in a money-pump. The most central premise of my argument holds that value is choice-guiding: If one object is more valuable than another, then it is not permitted to choose the less valuable object; and if two objects are equally valuable it is permitted to choose either of them; and if two objects are on a par or belong to the same clump it is also permitted to choose either of them.

In what follows, I shall discuss parity and clumpiness separately. It will be shown that the money-pump argument can be detailed in two different ways. The first version will be referred to as the sequential argument, whereas the second will be referred to as the non-sequential argument. The concept of parity is sensitive to both versions of the argument, whereas Hsieh's concept of clumpiness is only sensitive to the sequential argument. Section 2 is devoted to parity and the sequential argument, whereas section 3 is concerned with parity and the non-sequential argument. In section 4 I discuss the notion of clumpiness.

II. PARITY AND SEQUENTIAL CHOICE

The money-pump argument was initially invoked for justifying the assumption that rational preferences are transitive. However, when

[4] Nien-He Hsieh, 'Equality, Clumpiness and Incomparability', *Utilitas* 17 (2005), p. 184.

[5] Hsieh, 'Equality, Clumpiness and Incomparability', p. 182.

adopted for challenging the possibility of parity, its basic structure is as follows. Let x be on a par with y, and let the same be true of y and x^+, where x^+ is a slightly improved, strictly better version of x. Parity is a non-transitive relation, so even though x is on a par with y and y is on a par with x^+, it is not inconsistent to assume that x and x^+ are not on a par. Imagine that you are in possession of x^+, and that you are offered to swap x^+ for y. Since the two objects are on a par, it seems reasonable to assume that you are permitted to swap. So you swap, and get y. You are then offered to swap y for x, which you do, since they are on a par. Finally, you are offered to pay a small amount of money for swapping x for x^+. Since x^+ is strictly better than x, you accept the offer and end up where you started, the only difference being that you are now a little bit poorer. This is clearly irrational.

In order to spell out the money-pump argument in more detail, let $>$ denote strict betterness, and let $=$ denote equality of value. Parity will be denoted by the symbol $\cong$, i.e. $x \cong y$ if and only if x is on a par with y. We extend the standard notation of deontic logic by a temporal index. $P_t(x)$ means that it is permissible to choose x at time t. The first premise, holding that value is choice-guiding, can now be spelled out as follows.

Premise 1 In a pairwise choice between x and y at time t:

(i) If $x > y$, then $P_t(x) \wedge \neg P_t(y)$.
(ii) If $x = y$, then $P_t(x) \wedge P_t(y)$.
(iii) If $x \cong y$, then $P_t(x) \wedge P_t(y)$.

Some people may be inclined to argue that Premise 1 is incompatible with the so-called buck-passing account of value. On the buck-passing account, to be valuable is to have a set of properties that provides reasons for favouring or choosing the object in question.[6] Hence, it would be wrong to assume that value *determines* rational choice; it is rather the other way around. In response to this objection, it should be noted that Premise 1 is not supposed to be a metaphysical claim about the nature of value. It is just meant to be a claim about an alleged relationship between value and rational choice, which holds true no matter which metaphysical theory of value happens to be true. Therefore, even advocates of the buck-passing account may agree that the material implications (i)–(iii) are all true, since these principles tell us nothing about what value *is*. Premise 1 is thus acceptable no matter which theory of value one subscribes to.

Another objection, raised by Wlodek Rabinowicz, is to argue that in case x and y are on a par, it is neither permissible nor forbidden

[6] Thomas M. Scanlon, *What We Owe To Each Other* (Cambridge, Mass., 1998).

to choose any of the objects.[7] If two objects are on a par, there is a corresponding lack of precision on the normative level. Hence, Premise 1 fails. However, this objection is problematic. First of all, it forces us to give up one of the most fundamental assumptions of deontic logic, namely that every alternative action is either permissible or forbidden (but not both). Rabinowicz's revision of deontic logic is simply too drastic. Second, Rabinowicz's proposal is of little help for an agent wishing to find out what to do. The point is that value is supposed to be choice-guiding – this is why we care so much about value – but on Rabinowicz's account it is not. This makes his proposal somewhat unhelpful.

The second premise of the sequential argument is the principle of no payment. It holds that it is never permitted to pay for an object x, if x could have been obtained for free. This premise is not universally valid, since things might change over time. Though you now could get a copy of my PhD thesis for free, it might be rational to pay a huge amount for it in the future, if I become as famous as Jean-Paul Sartre. However, given that the principle of no payment is restricted to short periods of time, it is difficult to reject. For the sake of the argument, we assume that the principle of no payment holds at least in the following situation: You will shortly receive exactly one valuable object, x, y, or z. You know that these three are the only objects you can ever obtain; there are no better outcomes. You are then offered a sequence of pairwise choices between the three objects. Now, if it is permitted to choose x at one point during the course of the experiment, then it can hardly be permitted to choose a strictly worse outcome, $x - \varepsilon$, at a later point.

To 'pay' for an object x means that the agent gives up some small amount of value ε, not necessarily money, in exchange for x. The formula $x - \varepsilon$ is thus a metaphorical expression of this subtraction. Strictly speaking, $x - \varepsilon$ should be conceived of as an ordered pair $\langle x, -\varepsilon \rangle$. Consider the following premise, which holds for all x and t:

Premise 2 If $P_t(x)$, then $\neg P_{t+n}(x - \varepsilon)$ for all $n \geq 1$.

In addition to the premises stated above, the following technical assumption will also be made: For every pair of distinct objects x, y, if $x > y$ at t, then there exists some small amount of value ε such that $x - \varepsilon > y$ at t. Call this Assumption T.

We are now in a position to prove the following theorem:

Theorem 1 Premises 1 and 2 and Assumption T are logically inconsistent.

[7] In conversation, August 2005.

Proof. Suppose for *reductio* that x $\cong$ y and y $\cong$ z and x > z. Let t_1, t_2, t_3 be three points in time and suppose that the agent is offered a choice between x and y at t_1, and between y and z at t_2, and between z and x at t_3. It follows from Premise 1 that $P_{t1}(x)$ in the choice between x and y, since x $\cong$ y. Now consider the choice made at t_3. Assumption T guarantees that there is some small amount of value ε such that x − ε > z. Therefore, in a choice made at t_3 between z and − ε (rather than x), Premise 1 implies that $P_{t3}(x - \varepsilon)$. However, since $P_{t1}(x)$, as showed above, Premise 2 implies that $\neg P_{t3}(x - \varepsilon)$.

Theorem 1 is designed to show that there is no such thing as parity. Its basic message is that if parity exists, it has to be related to rational choice in one way or another. Value is choice-guiding, at least in the context considered here. But given Premise 1, which is arguably the correct way of linking value to rational choice, it follows that if parity exists, a rational agent could be exploited.

The argument considered here is parallel to the traditional money-pump argument for transitivity, showing that an agent whose strict preference is not transitive can be exploited by repeatedly offering to pay him a small amount for swapping one object for a better one. Some of the criticism raised against the traditional money-pump argument affects the new version of the argument as well. For example, Frederic Schick famously pointed out that a clever decision-maker facing a money-pump would see 'what is in store for him [and] reject the offer and thus stop the pump'.[8] Thus, it cannot be taken for granted that someone who knows that he will be offered to swap several times should be prepared to do so, since he can simply see what is going to happen.

Schick's idea can be rendered more precise by using backwards induction, as shown by McClennen.[9] However, Wlodek Rabinowicz has pointed out that even a decision-maker who reasons backwards, from the end node of the decision tree up to the top, and then decides what to do, can be exploited in a modified (more complex) money-pump.[10] It is beyond the scope of the present contribution to comment on Rabinowicz's version of the money-pump argument. Let me just point out that there seems to be another way of avoiding the criticism raised by Schick. Suppose that the agent has no reason to believe that he will be offered to make any more choices. Each new offer to swap comes as a surprise to him. All he knows is that x, y, and z are the possible outcomes (plus a finite number of ε:s). It still seems plausible to maintain that

[8] Frederick Schick, 'Dutch Bookies and Money Pumps', *The Journal of Philosophy* 83 (1986), p. 117.

[9] Edward F. McClennen, *Rationality and Dynamic Choice* (Cambridge, 1990).

[10] Wlodek Rabinowicz, 'Money Pump with Foresight', *Imperceptible Harms and Benefits*, ed. Michael J. Almeida (Dordrecht, 2000), pp. 123–54.

a reasonable theory of rational choice should guarantee that it is not permissible to choose (x - ε) at t_3 if x could have been chosen at t_1. If one knew from the beginning that one could choose x and stay with it, and that there was no other strictly better object, why should one then accept to end up with $x - \varepsilon$?

In a recent paper, Chang briefly comments on a (less detailed and logically inconclusive) version of the money-pump argument.[11] According to Chang, 'The rational permissibility of choosing either of two items on a par, then, must be constrained by one's other choices', and she goes on and claims that 'the sense in which it may be rationally impermissible to choose one of two items on a par depends on understanding the rationality of choice against a background of other choices'.[12] As far as I can see, Chang tries to make the same point as Schick: the rational agent will foresee what is going to happen, and should therefore refuse any offer to swap. But, again, as explained above, this reply is not relevant if we just add the assumption that the agent does not know, or has any reason to believe, that he might eventually be offered to swap. All that is known at the beginning of the experiment is that there are three objects x, y, z, and that you will get x if you do nothing. You are then offered to swap x for y. The offer comes as a surprise to you, and you know that you will never end up with something that is better than x (since neither y nor z is strictly preferred to x). However, you consider x and y to be on a par, so you are therefore permitted to swap. And so on.

III. PARITY AND NON-SEQUENTIAL CHOICES

The money-pump argument can be spelled out in an alternative, non-sequential way. In the non-sequential version, it is not assumed that the agent makes pairwise, sequential choices, nor does the argument rely on the principle of no payment. The set of alternatives is kept constant, and only one choice is made by the agent. Arguably, this is an improvement on previous pragmatic arguments proposed in the literature.[13]

Premise 1*

(i) If $x > y$, then $\neg Py$.
(ii) If $x = y$, then Px iff Py.
(iii) If $x \cong y$, then Px iff Py.

[11] Ruth Chang, 'Parity, Interval Value, and Choice', *Ethics* 115 (2005), pp. 346–7.
[12] Chang, 'Parity, Interval Value, and Choice', p. 347.
[13] See e.g. Ruth Chang, 'Introduction', *Incommensurability, Incomparability and Practical Reason* (Cambridge, Mass., 1997) and 'Parity, Interval Value, and Choice'.

Premise 1* is applicable to choices among any (finite) number of alternatives, whereas Premise 1 is only applicable to choices between exactly two objects. Premise 2 is not employed in the non-sequential argument. The following weak premise is sufficient for deriving a contradiction.

Premise 3 In every choice it is permissible to choose at least one alternative.

Theorem 2 Premise 1* and Premise 3 are logically inconsistent.

Proof. Suppose for *reductio* that the agent is offered a choice among three objects x, y, z, and that $x \cong y$ and $y \cong z$ and $x > z$. It follows from Premise 3 that at least one alternative is permissible. It cannot be z, because of Premise 1, part (i). Hence, at least one of x or y is permissible. However, in every possible circumstance, Premise 1, part (iii), guarantees that Px and Py, since $x \cong y$. Furthermore, since $y \cong z$, Premise 1 also implies that Pz. However, as noted above, $x > z$, so $\neg$Pz.

I am aware of two objections to the non-sequential argument. The target of both objections is Premise 1*. The first objection holds that even if two objects are on a par, it may nevertheless be permissible to choose only one of them.[14] For example, consider a choice among x, y, and z, in which $x \cong y$, and $y \cong z$. Also suppose that $x > z$. Then, it is permissible to choose y but not z, contrary to what is prescribed by Premise 1*, according to the advocates of this objection. The reason why it is not permissible to choose z is that there is an object that is strictly better. The fact that y is permissible and y is on a par with z does not imply that z is permissible. A more holistic approach is needed for determining the deontic status of an alternative – call this the 'holistic' argument.

A problem with the holistic argument is that it violates the principle of irrelevant alternatives, or at least its 'deontic spirit'.[15] Suppose that y and z are the *only* alternatives available. Then, since they are on a par, they are both permissible. So how could the introduction of a third alternative affect the relative ranking of y and z? In order to spell out this argument in more detail, it is helpful to make a sharp distinction between evaluative and deontic properties. An object's evaluative properties determine how good or bad the object is. Its deontic properties tell us whether the object is permissible to choose. I shall assume that deontic properties supervene on evaluative

[14] This objection was suggested to me by Eric Carlsson and Tor Sandqvist in conversation.

[15] See e.g. Amartya Sen, 'Internal Consistency of Choice', *Econometrica* 61 (1993), pp. 495–521.

properties in the following sense: An evaluative ranking of a set of objects, in which all objects are listed from the best to the worst, determines the deontic properties of the objects. Hence, if two objects are on a par on the evaluative scale, and therefore have the same deontic properties, the addition of a third object cannot affect the deontic property of one of the original objects, unless that of the other object is also affected. To deny this principle would be very metaphysically odd.

According to the second objection to Premise 1*, due to Chang, there might exist different kinds of permissibility. The kind of permissibility 'of choosing between items that are equally good is a different kind... from the "permissibility" of choosing between items that are on a par'.[16] Chang admits that this way of reasoning implies drastic revisions of traditional decision theory and deontic logic. However, according to Chang, 'our formalizations should follow our philosophy.... if they're too crude, we should reject them'.[17] I am inclined to agree with Chang, at least in principle. She is certainly right in claiming that our formalizations should follow our philosophy. However, as far as I know, no convincing argument has yet been given for thinking that there really exist different kinds of permissibility. So far, this looks more like an *ad hoc* hypothesis, introduced merely for saving the notion of parity. We have to be presented with more evidence for Chang's view before it can fully assessed, and eventually accepted.

IV. CLUMPINESS

In the foregoing sections it has been shown that reasonable ways of linking parity to rational choice lead to inconsistency. An analogous argument could also be employed for challenging Hsieh's concept of clumpiness.

By definition, all objects that belong to the same clump are equally good, and no object is a member of more than one clump, relative to a given 'resolution' of a comparison.[18] Hsieh mentions the example of grading student essays.[19] All essays that belong to the same clump, say grade B, are by definition equally as good, as long as the resolution of the comparison is kept constant. If the resolution of the comparison is increased, it might be meaningful to distinguish between essays belonging to clumps corresponding to B^+, B, and B^-, etc. However, the

[16] Email correspondence, 11 September 2005.
[17] Email correspondence, 12 September 2005.
[18] The condition that no object is a member of more than one clump is not explicitly mentioned by Hsieh, but I shall nevertheless assume that he accepts this condition – otherwise it would of course be trivial to construct a money-pump.
[19] Hsieh, 'Equality, Clumpiness, and Incomparability', p. 184.

resolution of a comparison is not to be chosen arbitrarily. According to Hsieh, 'the resolution of the comparisons involved is determined by the purpose for which the comparisons are made'.[20] Hence, different levels of resolution cannot be adopted in one and the same comparison; only one level of resolution is applicable in each comparison made by the agent.

Now consider the sequential argument. Suppose Hsieh wishes to select the best student essay from a set of three essays x, y and z. Let us also suppose that if the comparison between the essays is made at a high resolution, then x is strictly better than z, but if the comparison is made at a low resolution all three essays belong to the same clump. At t_0 Hsieh holds essay x in his left hand. He is then offered a choice between x and y at t_1, and if he swaps he will (which will come as a surprise to him) be offered a choice between y and z at t_2, and if he swaps again he will finally be offered a choice between z and x at t_3. The circumstances of the choices happen to be such that at t_1 and t_2 the level of resolution ought to be low. Therefore, x and y belong to the same clump, so Hsieh is therefore permitted to swap, and the same holds true of y and z. However, at t_3, the circumstances happen to be slightly different, so now a high level of resolution is required. Hence, in the choice between z and x, the latter essay is strictly better and Hsieh would therefore be prepared to pay some small amount of money for swapping back to x. Arguably, this is irrational, for the reason mentioned in Section 2: he knew from the beginning that no matter which level of resolution was used, no other essay would ever turn out be better than x – so why adopt a notion of value that allows one to start swapping?

What about the non-sequential argument? Arguably, this argument cannot be employed for challenging the possibility of clumpiness. This is because the notion of clumpiness makes it impossible to construct a situation in which x, y and z belong to the same clump at the same time as x is strictly better than z. Therefore, if one could think of some reason for claiming that the non-sequential argument, but not the sequential one, is unsound, it could perhaps be argued that clumpiness is after all a more reasonable concept than parity. However, I am not aware of any such reason.[21]

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[20] Hsieh, 'Equality, Clumpiness, and Incomparability', p. 186.

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